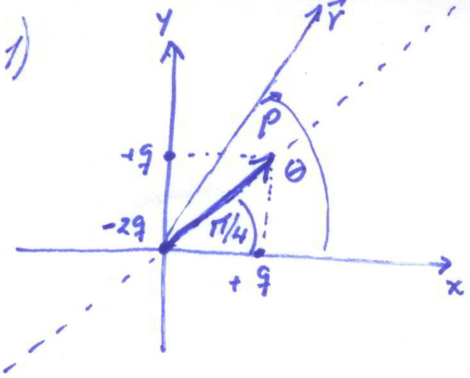
1)



For $|\underline{r}| >> l$ we can consider only the first two terms in the multipole expansion:

$$V(\underline{r}) \approx V_{mon}(\underline{r}) + V_{dip}(\underline{r})$$

$$V_{mon}(\underline{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$ where $Q = \sum_i q_i = 0$ in our case so:

$$V_{mon}(\underline{r}) = 0$$

and

$$V_{dip}(\underline{r}) = \frac{1}{4\pi\epsilon_0}\frac{\underline{p}\cdot\hat{r}}{r^2}$$ where $\underline{p} = \sum_i q_i \underline{r}_i' = ql(\hat{i}+\hat{j})$ in our case

$$V_{dip}(\underline{r}) = \frac{1}{4\pi\epsilon_0}\frac{ql}{r^2}(\cos\theta + \sin\theta)$$ this can also be written as:

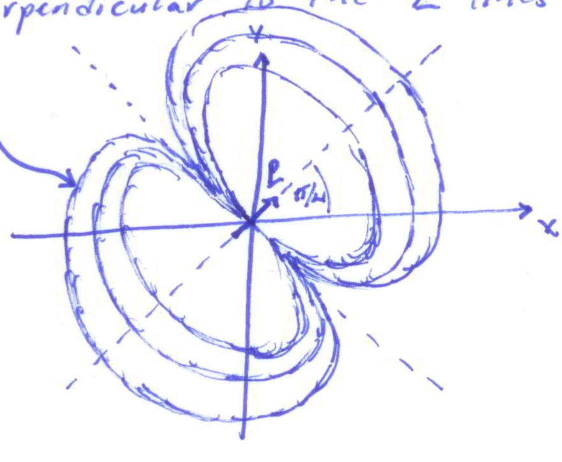
$$V_{dip}(\underline{r}) = \frac{1}{4\pi\epsilon_0}\frac{\sqrt{2}\,ql}{r^2}\cos\left(\theta - \frac{\pi}{4}\right)$$

So, finally:

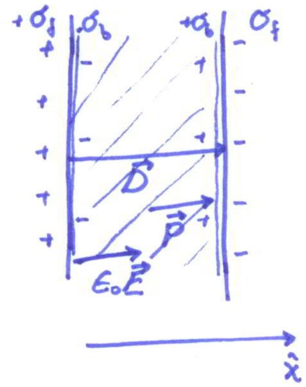
$$\underline{\underline{V(\underline{r}) = \frac{1}{4\pi\epsilon_0}\frac{\sqrt{2}\,ql}{r^2}\cos\left(\theta - \frac{\pi}{4}\right)}}$$

b) The curves $V(\underline{r})$ = constant are perpendicular to the $\vec{E}$ lines

$V(\underline{r})$ = constant $\Rightarrow r = \lambda\sqrt{\cos\left(\theta - \frac{\pi}{4}\right)}$



2-)



$$\vec{P} = \vec{D} - \epsilon_0 \vec{E}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon} \qquad \vec{D} = \sigma_f \hat{x} = \frac{Q}{A} \hat{x}$$

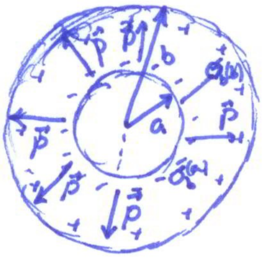
$$\vec{P} = \vec{D}\left(1 - \frac{\epsilon_0}{\epsilon}\right)$$

$$\vec{P} = \frac{Q}{A}\left(1 - \frac{\epsilon_0}{\epsilon}\right) \hat{x}$$

$$\sigma_{bound} = P$$

$$\sigma_{bound} = \frac{Q}{A}\left(1 - \frac{\epsilon_0}{\epsilon}\right)$$

3)



$$\underline{P}(\underline{r}) = \frac{K}{r}\hat{r}$$

$$\sigma_{bound} = |\underline{P}| \cdot \hat{r}\cdot\hat{n} \qquad \rho_{bound} = -\nabla\cdot\underline{P} = -\frac{K}{r^2} \quad \text{for } a<r<b \text{, 0 otherwise}$$

In the inner surface $r=a$ and $\hat{r}\cdot\hat{n} = -1$

$$\sigma_{bound}(a) = -\frac{K}{a}$$

In the outer surface $r=b$ and $\hat{r}\cdot\hat{n} = 1$

$$\sigma_{bound}(b) = \frac{K}{b}$$

For $r<a$

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enclosed}}{\epsilon_0} = 0 \quad \text{so}$$

$$\underline{E} = \underline{0} \qquad a > r$$

<u>For $a<r<b$</u>

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enclosed}}{\epsilon_0} = \frac{1}{\epsilon_0}\left(\int_{r=a}\sigma_{bound}(a)\cdot da + \int_a^r \rho_{bound}\, 4\pi r'^2 dr'\right)$$

$$E\cdot\cancel{4\pi} r^2 = \frac{1}{\epsilon_0}\left(-\frac{K}{a}\cdot\cancel{4\pi}a^2 + \int_a^r\left(-\frac{K}{\cancel{r'^2}}\right)\cancel{4\pi r'^2}dr'\right)$$

$$E = \frac{1}{\epsilon_0 r^2}\left(-Ka - K(r-a)\right) = -\frac{K}{\epsilon_0}\,\frac{\cancel{a}+r-\cancel{a}}{r^2}$$

$$\underline{E} = -\frac{K}{\epsilon_0 r}\hat{r} \qquad a<r<b$$

<u>For $r > b$</u>

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enclosed}}{\epsilon_0} = \frac{1}{\epsilon_0}\left(\int_{r=a}\sigma_{bound}(a)\,da + \int_a^b \rho_{bound}\cdot 4\pi r'^2 dr' + \int_{r=b}\sigma_{bound}(b)\,da\right)$$

$$E\cdot 4\pi r^2 = \frac{1}{\epsilon_0}\left(-\frac{K}{a}\cdot 4\pi a^2 - K\cdot 4\pi(b-a) + \frac{K}{b}\cdot 4\pi b^2\right) = 0 \text{, so } \underline{E} = \underline{0} \qquad b<r$$